Horizon Community College
Numeracy
Corporate Methods Guide

Contents

Order of Operations

Use of corporate language: BIDMAS will only be taught as BIDMAS. The I stands for indices. BODMAS is not to be used.

Possible Misconceptions:

- Students must understand that the division and multiplication can be completed in any order, as can an addition and subtraction.
- In the following example $(x+4)^3$ students may attempt to add the x and the 4 forgetting that the expression is already simplified fully. They need to recognise $(x+4)^3 = (x+4)(x+4)(x+4)$

Presenting Numbers

Method: When writing numbers students are to be taught to use no commas in big numbers e.g. 1 000 000. A small space will be used where the comma once was. This will stop pupils confusing a decimal point and a comma where in Europe a comma is used to denote a decimal point.
Example:

£1 350 000 (correct)

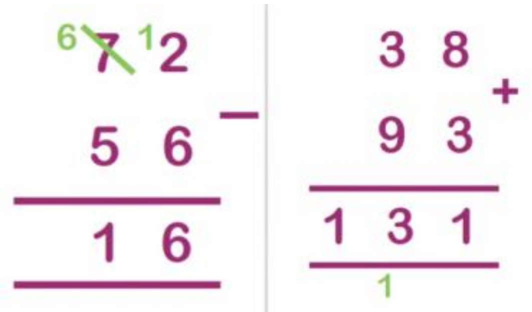
£1,350,000 (incorrect)

Four Operations

Addition and Subtraction

Method: Addition and subtraction of numbers including decimals should be done in columns where adding takes place from the right hand column first.

Example:

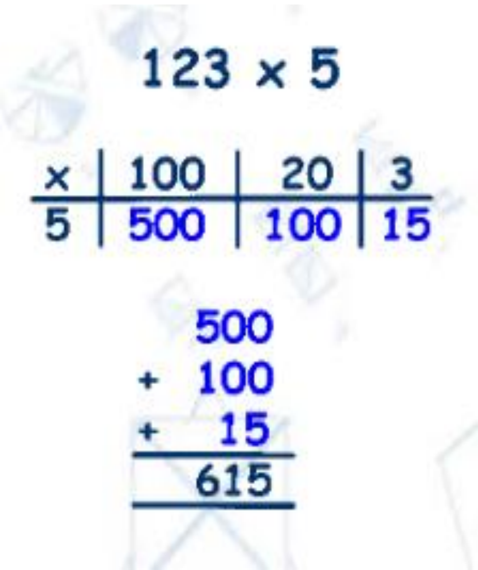


Use of corporate language: For subtraction, the language should be around exchanging between columns not borrowing.

Long Multiplication

Method: For long multiplication, the grid method is used. Although many students will know alternative methods the grid method prepares them for expanding and factorising expressions at a later point.

Example:

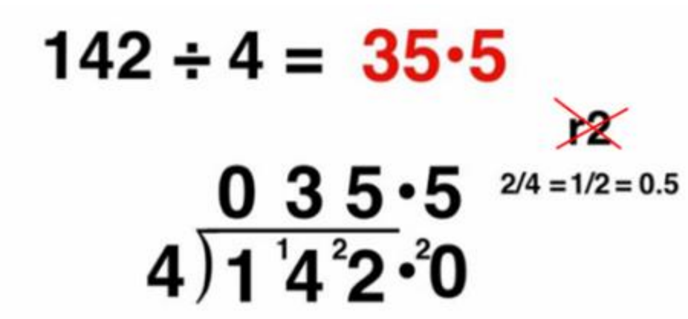


Possible Misconceptions: Students may expand 123 to simply 1, 2 and 3 rather than 100, 20 and 3. When adding the result, they need to ensure the numbers are correctly lined up to avoid place value calculation errors.

Short Division

Method: For short division, the box should be set up with the option of the multiples of the divisor written for help. When using decimals, the decimal points should line up. *Students should be able to calculate a decimal answer to any division, rather than leaving a remainder.* They also need to be able to give an answer as a mixed number, using their remainder as the numerator and the divisor as the denominator.

Example:

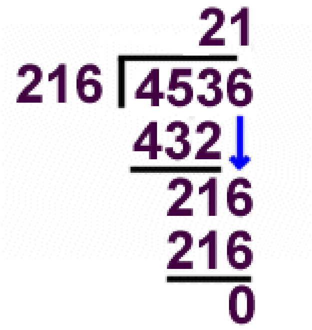


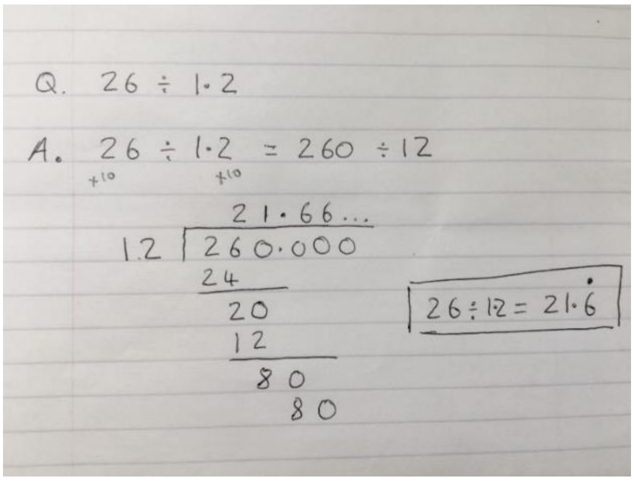
Use of corporate language: Students may have been introduced to sort division as the 'Bus Stop' methods, refrain from using this term and embed the phrase short division.

Long Division

Method: Long division with decimals should be taught using equivalent divisions. For example, 26/1.2 can be multiplied by ten to give 260/12.

Example:



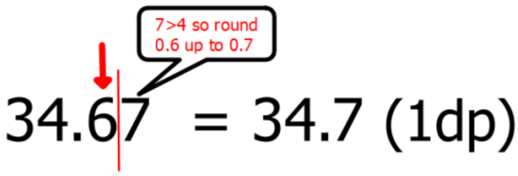


Rounding

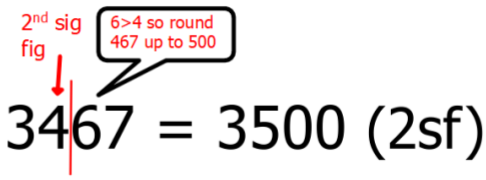
Method: Students need to explicitly identify the place value holder the question is asking for a number to be rounded to. Students need to do this by drawing an arrow above the number, pointing to the number and state the place value of this number. Students then need to draw a line after this number and begin the rounding process. Ensure it is stressed to students the importance of this topic as rounding carries marks at GCSE.

Example:

Round 34.67 to 1 dp



Round 3467 to 2 sf



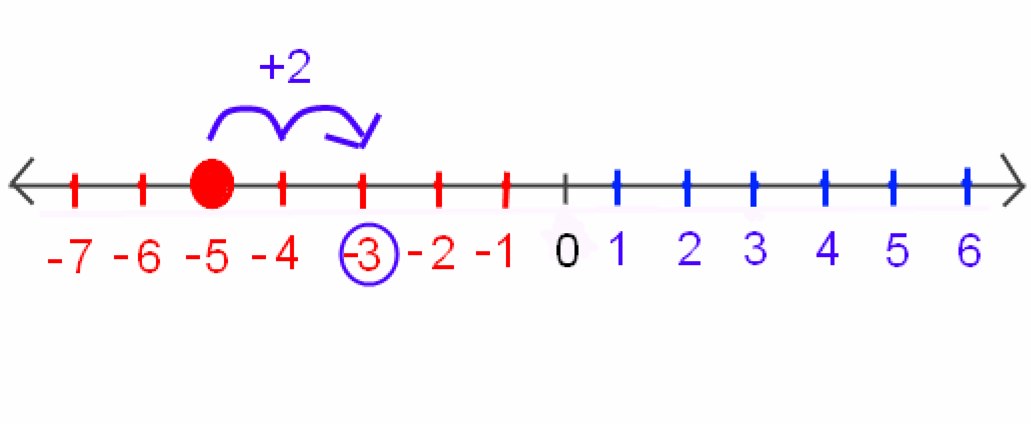
Use of corporate language: Ensure students use the terminology of upper and lower bounds and error intervals.

Possible Misconceptions: Students are likely to get rounding to decimal points and significant figures mixed up, emphasise the language used and expose them to decimal numbers that need to be rounded to significant figures. Watch out for students forgetting to replace digits with 0s when dealing with significant figures, ask them if rounding 5423 to 54 makes mathematical sense.

Negative Numbers

Method: When adding and subtracting negative numbers there should be a starting point (first number) to help follow on a number line, and an operator. Students should be exposed to sums with and without brackets 3-(-2) and 3--2. The analogy to be used with all students is positive and negative behaviour points, and net points. E.g. if I were to delete a negative behaviour point what would happen to the net points? It would increase.

Example: See the use of the number line below for the sum -5 + 2, negative 5 has been used as a starting point and the operation (add 2) has been completed to lead the student to an answer.



Use of corporate language: When talking about negative numbers teachers will only refer to '-5' as negative 5, never minus 5, students could confuse the phrase 'minus 5' with an operation rather than a term. Some pupils may believe that -6 is greater than -3. For this reason, ensure pupils avoid saying "bigger than" or "smaller than" and use the phrases "greater than" or "less than."

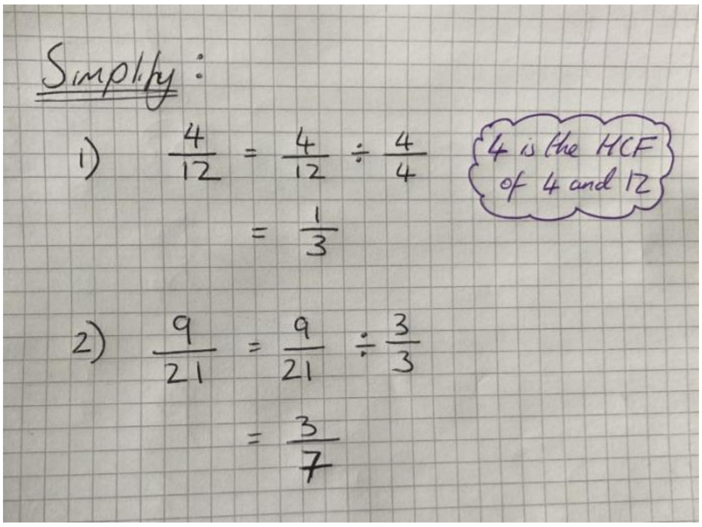
Possible Misconceptions: Some pupils may write statements such as 140 - 190 = 50. When subtracting mentally some pupils may deal with columns separately and not combine correctly, e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So, the answer is 166. Another common misconception is students stating "two negative make a positive" when trying to add two negative numbers together, please be careful with your use of language in the exposition.

Fractions

Simplifying Fractions

Method: When simplifying fractions or doing any work with changing the appearance of a fraction using equivalence, ensure everyone is explaining it and insisting on it being written as a multiplication/division of a form of 1 i.e. 3/3 not just x3 written next to the numerator and x3 written next to the denominator.

Example:

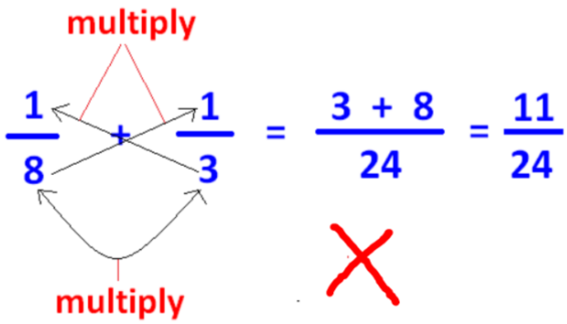


Adding and Subtracting Fractions

Method: When adding and subtracting fractions the denominators should be made equivalent. Teachers should teach students to identify the LCM of the two fractions and not to cross multiply.

Example:

$$\frac{3}{4} + \frac{1}{3} = \frac{3 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4}$$
$$= \frac{9}{12} + \frac{4}{12}$$
$$= \frac{13}{12} = 1\frac{1}{12}$$



Use of corporate language: Teachers should always refer to the denominator and numerator, correcting students if they use the term 'bottom or top number.' Students should be able to understand LCM (Lowest Common Multiple) and confident with using the term equivalency.

Possible Misconceptions: Students may simply add the numerator and the denominator, e.g 1/4 + 1/2 = 2/6. Students do not always give the answer in its lowest terms.

Multiplying Fractions

Method: When multiplying a fraction by an integer, students must rewrite the questions to the integer is in fraction form. When multiplying two fractions together students multiply both numerators by each other followed by both denominators by each other. Students should simplify their answers, leaving as a mixed number unless otherwise stated.

Example:

Integer x Fraction

$$\frac{4}{9}\times 3$$
$$= \frac{4}{9}\times\frac{3}{1}$$
$$= \frac{4\times 3}{9\times 1}$$
$$= \frac{12}{9} \quad = 1\frac{1}{3}$$

Fraction x Fraction

$$\frac{2}{5} \times \frac{6}{7} = \frac{2 \times 6}{5 \times 7} = \frac{12}{35}$$

$$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12} = \frac{1}{6}$$

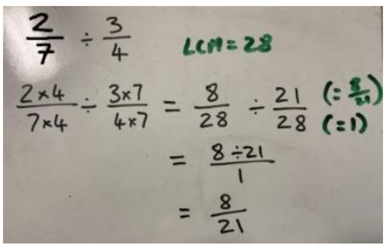
Use of corporate language: Teachers should always refer to the denominator and numerator, correcting students if they use the term 'bottom or top number.' Teachers should always refer to the operation a 'multiply' never 'times.'

Possible Misconceptions: Some pupils may write 1/3 instead of 3/1 when converting an integer into a fraction. Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly).

Dividing Fractions

Method: Teachers will not be using the KFC (Keep, Flip, Change). Instead students will be taught how to equate denominators and then divide numerators and denominators.

Example:



Use of corporate language: Teachers should always refer to the denominator and numerator, correcting students if they use the term 'bottom or top number.'

Possible Misconceptions: Students may need clarification on why the denominators will divide to 1.

Indices

Multiplying and Dividing Indices

Method: When multiplying/dividing the same base number raised to indices the teacher should try and draw out the rule of adding/subtracting indices from the class. Students should already be aware that $2^3 = 2 \times 2 \times 2$ and therefore should be allowed to make the connection that $2^3 \times 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ is equivalent to 2^8. This could be scaffolded with discussion and board work to draw this out of students.

Students should be exposed to generalisation of Index laws e.g. $\mathbf{a^b \times a^c = a^{b+c}}$

Examples:

$$2^4 \times 2^3$$

$$\underbrace{2 \times 2 \times 2 \times 2}_{} \times \underbrace{2 \times 2 \times 2}_{}$$

$$= 2^7$$

$$a^x \times a^y = a^{x+y}$$

Division of indices with same base:

$$a^m \div a^n = a^{m-n}$$

$$b^{m-n} = b^m \div b^n$$

Example:

$$\frac{c^9}{c^4} = c^{9-4} = c^5$$

$$3^{x-2} = \frac{3^x}{3^2}$$

$$\frac{\cancel{4}p^2}{\cancel{12}p^5} = \frac{p^{2-5}}{3} = \frac{1}{3}p^{-3} = \frac{1}{3p^3}$$

Use of corporate language: Teachers should refer to the base number and its index. E.g. in 2^3 the base would be 2 and the index would be 3. The plural of index is indices and when required to raise an index to another index this can be referred to as raising a power to another power.

Possible Misconceptions: In examples like $2^4 \times 2$ students often fail to realise that $2 = 2^1$ This should be explicitly discussed in the exposition.

Raising Indices to a Power

Method: $(k^3)^2$ means that k^3 is to be squared, or multiplied by k^3 again, show students that by raising to a further power of two k^3 is being squared, this should be written out as such $(k^3)^2 = k^3 \times k^3 = k^6$ in order for students to make links to the index law of multiplying indices with the same base numbers.

Example:

Raising an index to a power

$$(a^m)^n = a^{mn}$$

$$b^{mn} = (b^m)^n$$

EXAMPLE:

$$(b^4)^3 = b^{4\times 3} = b^{12}$$

$$(3^2)^3 = 3^{2\times 3} = 3^6$$

$$(2^x)^2 = 2^{2x}$$

$$(2^{y+1})^3 = 2^{3y+3}$$

$$3^{2c} = (3^c)^2$$

Use of corporate language: Teachers should refer to the base number and its index. E.g. in 2^3 the base would be 2 and the index would be 3. When required to raise an index to another index this can be referred to as raising a power to another power.

Possible Misconceptions: The most common misconception is for students to add the indices rather than multiply them. In the event of a question such as $(2x^3)^4$ they mistake 2 a part of the base number leaving their answer as $2x^{12}$ rather that realising they must also raise 2 to the power of 4. Meaning the correct answer should be $16x^{12}$. Examples of this should be given in the exposition.

Negative Indices

Method: Introduce negative indices using an explanation with the index laws for division and subtracting the powers.

Example:

Simplify $d^4 \div d^5$.

Using index laws for division, subtract the powers.

$d^4 \div d^5 = d^{4-5} = d^{-1}$. This is an example of a **negative index**.

But $d^4 \div d^5$ also equals $\frac{d \times d \times d \times d}{d \times d \times d \times d \times d}$.

Cancelling **common factors** gives $\frac{\not{d} \times \not{d} \times \not{d} \times \not{d}}{\not{d} \times \not{d} \times \not{d} \times \not{d} \times d}$, which gives $d^4 \div d^5 = \frac{1}{d}$.

So $d^{-1} = \frac{1}{d}$.

The rule for negative indices is $a^{-m} = \frac{1}{a^m}$

Possible Misconceptions: Students are likely to associate a negative index with multiplying the base number by the negative e.g. $4^{-3} = -12$

Fractional Indices

Method: An example of a fractional index is $g^{1/3}$. The **denominator** of the fraction is the **root** of the number or letter, and the numerator of the fraction is the power to raise the answer to. Students must be exposed to examples where the numerator is >1 and where the fractional indices is negative. Students should be encouraged

Example:

$a^{\frac{1}{2}} = \sqrt{a}$, $a^{\frac{1}{3}} = \sqrt[3]{a}$ and so on.

By using index laws for multiplication from earlier it is clear to see that:

$g^{\frac{1}{2}} \times g^{\frac{1}{2}} = g^1$

Therefore: $g^{\frac{1}{2}} = \sqrt{g}$

$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$

Use of corporate language: Teachers should refer to the denominator as the root and the numerators at the power the base is to be raised to.

Possible Misconceptions: A common misconception is for students to mix up the purpose of the numerators and denominator. If a base is raised to a power of ½ students will often try to halve the number.

Standard Form

Standard form is a system of writing numbers which can be particularly useful for working with very large or very small numbers. It is based on using powers of 10 to express how big or small a number is.

Scientists use standard form when working with the speed of light and distances between galaxies, which can be enormous. The size of bacteria or atoms may also be referred to in standard form as they are so tiny. Standard form is also sometimes referred to as **scientific notation**.

Standard form to Ordinary Numbers

Method: Standard form uses the fact that the decimal place value system is based on powers of 10. A number written in standard form is presented as A x 10^n, where A is a number bigger than or equal to 1 and less than 10. n can be any positive or negative whole number. When converting numbers from standard from to ordinary numbers students need to understand it is a multiplication process; teachers should refer to digits being repeatedly multiplied or divided by powers of 10.

Example:

Write 1.34 x 10^3 as an ordinary number?

1.34×10^3

$= 1.34 \times 10 \times 10 \times 10$

$= 1\ 340$

Use of corporate language: The description 'standard form' is always used instead of 'scientific notation' or 'standard index form'.

Possible Misconceptions: When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros, e.g. $4 \times 10^5 = 400\ 000$ so $4.2 \times 10^5 = 4\ 200\ 000$.

34×10^7 is not in standard form as the first number is not between 1 and 10. To correct this, divide 34 by 10. To balance out the division of 10, multiply the second part by 10, which gives 10^8.

34×10^7 and 3.4×10^8 are identical in value but only the second is written in standard form.

Ordinary Numbers to Standard form

Method: Standard form uses the fact that the decimal place value system is based on powers of 10. A number written in standard form is presented as A x 10^n, where A is a number bigger than or equal to 1 and less than 10. n can be any positive or negative whole number.

Example:

What is 87 000 in standard form?

87 000 can be written as 8.7×10,000.

10 000=10×10×10×10=104

So, 87 000=8.7×104.

Use of corporate language: When converting from numbers given in ordinary form to numbers in standard form teachers should refer to the digits being repeatedly multiplied or divided by 10, i.e. the decimal point remains fixed.

Possible Misconceptions: When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros, e.g. 4×10^5 = 400 000 so 4.2×10^5 = 4 200 000.

Adding and Subtracting numbers in Standard Form

Method: A common method is to convert the numbers from standard from to ordinary numbers, complete the calculation and convert the answer back into standard from however this method increase the opportunities for students to make arithmetic mistakes. Alternatively, students should be taught to alter the numbers to that 10 is raised to the same power, complete the calculation and convert the answers back to standard form if needed.

Example:

Calculate $(2.4 \times 10^5) + (1.68 \times 10^2)$

Write the 1.68×10^2 as 10^5

$$1.68 \times 10^2 = 0.00168 \times 10^5$$

Add the two decimals

$$\begin{array}{r} 0.00168 \times 10^5 \\ + \underline{2.40000 \times 10^5} \\ 2.40168 \times 10^5 \end{array}$$

Use of corporate language: Refer to 10 as the base number of the indices as this will prompt them to connect with the laws of indices.

Possible Misconceptions: Students may have difficulty converting number in standard form to numbers raised to a different power of 10. This will need to be practiced prior to attempting this topic.

Multiplying and Dividing numbers in Standard Form

Method: Then multiplying and dividing number in stander form the laws of indices should be applied. Students should be taught to multiply or divide the numbers first then apply the laws of indices to the powers of 10.

Example:

Calculate $(3\times10^3) \times (3\times10^9)$

Multiply the first numbers: $3\times3=9$.

Apply the index law on the powers of 10: $10^3\times10^9=10^{3+9}=10^{12}$

$(3\times10^3) \times (3\times10^9) = 9\times10^{12}$

Note - Sometimes students may have to readjust their answers after the calculation in order leave it in standard form.

Calculate $(2 \times10^3) \div (8 \times10^9)$

$2 \div 8 = 0.25$

$10^3 \div 10^9 = 10^{-7}$

$(2 \times10^3) \div (8 \times10^9) = 0.25 \text{ x } 10^{-7}$

$= 2.5 \text{ x } 10^{-6}$ (The number must be left in standard from)

Use of corporate language: Refer to 10 as the base number of the indices as this will prompt them to connect with the laws of indices.

Possible Misconceptions: Students may forget to leave their answer in standard from. It is also common for student to forget the laws of indices and multiply the powers of 10 instead of adding them.

Percentages of an Amount (Calculator)

A percentage is a proportion that shows a number as parts per hundred. The symbol '%' means 'per cent'. 9% means 9 out of every 100, or 9/100. Students should know this and be able to articulate it.

Students also need to know that "out of" means divide and "of" means multiply.

Method: When using a calculator, students need to be able identify the percentage multiplier and use this to calculate the percentage. As students are being asked to find the percentage "of" and amount teacher should emphasise that this involves a multiplicative calculation.

Example:

Example. 7% of 48

The multiplier for this percentage is 0.07 (not 0.7 as some people might think)

So 7% of 48 = 0.07 x 48

= 3.36

Use of corporate language: Students should understand and confidently be able to use the word multiplier.

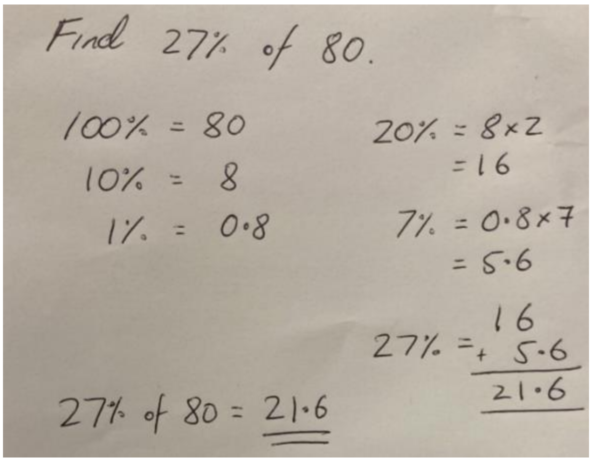
Possible Misconceptions: The most common misconceptions will occur when students are converting percentages into multipliers, particularly if the percentage is a single digit or decimal.

Percentages of an Amount (Non-calculator)

A percentage is a proportion that shows a number as parts per hundred. The symbol '%' means 'per cent'. 9% means 9 out of every 100, or 9/100. Students should know this and be able to articulate it.

Method: When calculating a % without a calculator ensure students use a build-up method to work it out. Students should be taught how to calculate 10%, 5% and 1% in order to 'build up' other percentages.

Example:



Possible Misconceptions: When trying to find 5% students will often divide the original number by 5 or to find 20% will divide by 20. Always get students to check if their answers make sense.

Percentage Change

Method: It is possible to look at the difference between two numbers and work out the percentage increase or the percentage decrease. This is known as percentage change. Often goods are bought for one price and then sold on for another. The percentage change can be calculated to find out the profit or loss an item has made.

Percentage change is calculated by dividing the difference between the two amounts by the original amount and multiplying by 100.

Percentage Change = (Difference ÷ Original) x 100

The difference is calculated by subtracting the new value from the original value.

Example:

Sheila bought a car for £1 560. After replacing the engine, she sold it for £2 760. Calculate the percentage change?

Percentage change (increase) $= \frac{change}{original\ quantity} \times 100\%$

$= \frac{£1,200}{£1,560} \times 100\%$

Percentage change =76.9%

Possible Misconceptions: A common misconception is for students to divide the original quantity by the change.

Compound Interest and Depreciation

Method: When calculating compound interest make sure students use the multiplier method. In order to calculate the compound interest, the students must first identify the multiplier and raise it to the power of the number of years the interest is for.

An example of finding the multiplier: If a bank decides to pay 5% interest on savings in a bank account then a 5% increase on the original balance in a bank would mean there is now 105% in the bank. This is the same as **1.05** as a decimal so this is the **multiplier**. Ensure students are exposed to example of depreciation. A car losing 8% of its value each year means that by the end of the year it is worth 92% of its original value, therefore the multiplier for this problem would be 0.92.

Example:

Calculate the interest on borrowing £40 for 3 years if the compound interest rate is 5% per year.

Multiplier – 1.05

Years – 3

$40 \times 1.05^3 = £46.31$

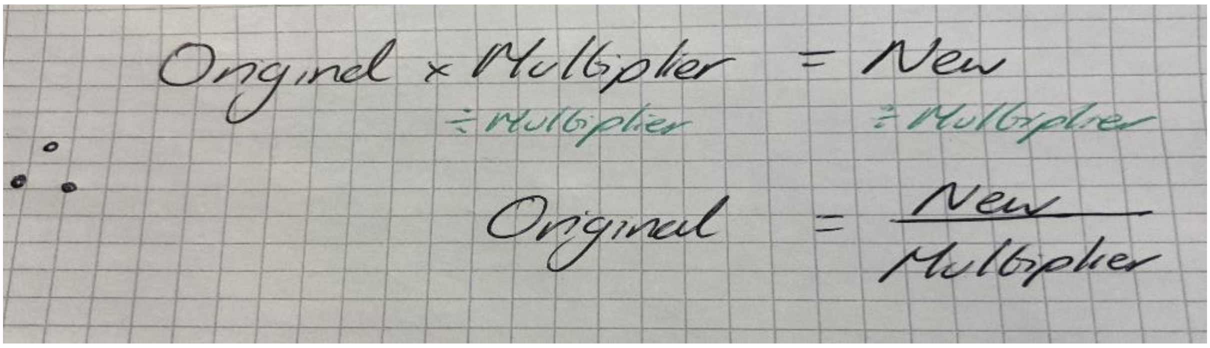
Use of corporate language: Make sure students are confident using the word multiplier. Students should be made aware that interest can be both paid and received depending if a monetary value is debt or savings respectfully.

Possible Misconceptions: Some pupils may think that the amount created by increasing a quantity by 5% repeated four times is the same as increasing the quantity by 5% and multiplying that amount by 4, using the multiplier and index method should prevent this. Some pupils may think the percentage multiplier for a 20% increase (or decrease) is 0.2

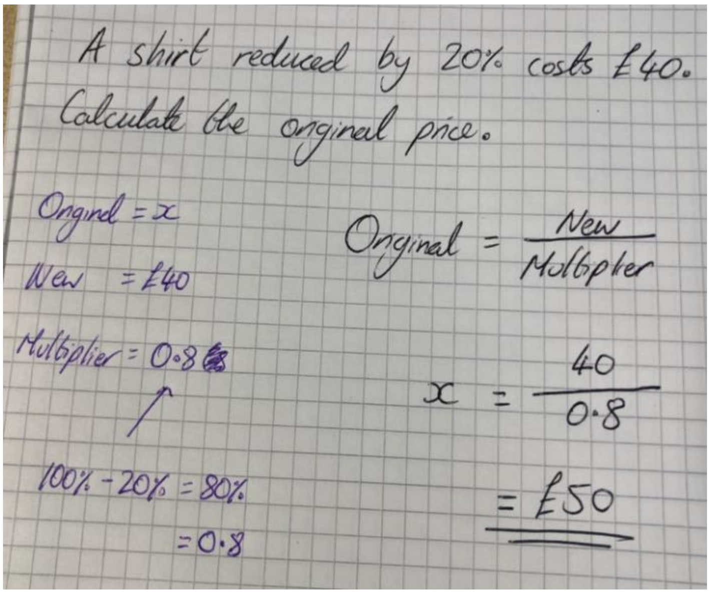
Reverse Percentages

Method: Reverse percentages help us to work out the original price or value of an item after it has been increased or decreased in value, for example, following a price increase or a sale. When calculating reverse percentages make sure students calculate the multiplier first and set up their equation, then divide through by the multiplier. Teachers may have previously taught students reverse percentages using the unitary method however students as HCC should be familiar using the multiplier to increase or decrease by a percentage and therefore teaching reverse percentage using the multiplier ensure continuity.

When introducing the concept, it is good practice to demonstrate the rearrangement of the equation used for a percentage increase/decrease in the exposition. See below.



Example:



Use of corporate language: Teachers should continually refer to the multiplier.

Possible Misconceptions: When being asked to find the original price of an item that has been reduced by 20% a common misconception is to work out 20% of the new price and add it on. This misconception should be explicitly referred to.

Compound Measure

Method: Compound measures are types of measure that involve two or more different units. Examples of compound measures include m/s, g/cm^3, population per km^2 and miles per gallon. As students have already studied using and rearranging formulae they should calculate compound units formally using one formula and then re-arranging it. When students are calculating compound units they must be careful with the units they write in their answer- when measuring the density of an object that has a mass measured in grams and a volume in cm^3 the answer must be left in terms of g/cm^3.

Example:

Speech is a compound measure, because it is calculated from two other measurements, *distance* and *time*.

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

The units of your answer will depend on the units of the question.

An athlete runs 100 metres in 20 seconds.

$$\text{average speed} = \frac{100\text{ m}}{20\text{ s}} = 0.5\text{ m/s}$$

Use of corporate language:

Possible Misconceptions: If students have had prior exposure to formula triangles they may confuse S=D/T and D=M/V as both involve a D. Students may have trouble leaving their answers in the correct units, teachers need to be explicit when teaching this.

Ratio

A ratio shows how much of one thing there is compared to another. Ratios are usually written in the form a:b.

If you are making orange squash and you mix one part orange to four parts water, then the ratio of orange to water will be 1:4 (1 to 4).

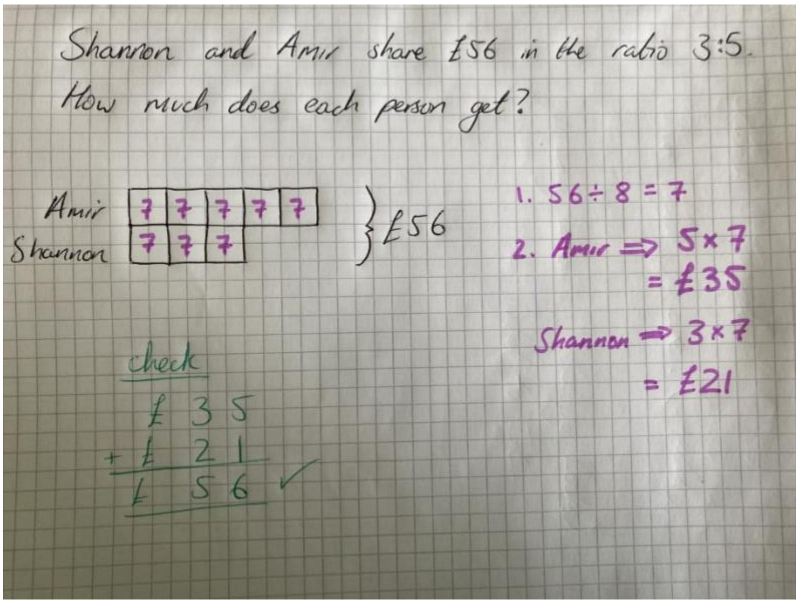
The order in which a ratio is stated is important. Changing the order of the numbers in a ratio changes the proportions.

Dividing a quantity into a ratio

Method: Teachers will take a common approach to teaching our students how to solve ratio problems - the bar method. To divide a quantity into a ratio the first step is to identify the total parts. A bar should then be drawn and split into equal sixed boxes, emphasising the value of each box is equal. In the example below Amir has 5 parts and Shannon has 3 parts, stack the bars on top of one another to prevent confusion- this will help students understand when only part of the total quantity is known. Show the students that the total quantity is divided by the total number of boxes; the result is the value of each part (or box). The result is them multiplied by the share of the ratio Amir and Shannon get.

Students must check their answer by adding each share and ensuring it equals the total quantity.

Example:

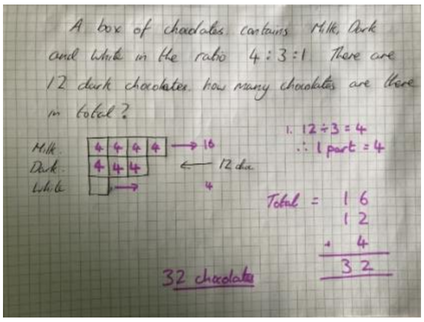


Possible Misconceptions: Students often mix up the ratio for example they may assign 3 parts to Amir and 5 parts to Shannon, language is important. Students also find 3-part ratios difficult.

Finding one quantity when another is known

Method: Teachers will take a common approach to teaching our students how to solve ratio problems - the bar method. When only a part of a total quantity is known the bar model important for students understanding. In the example below milk, dark and white chocolates are in a ratio of 4:3:1 respectfully. The number of dark chocolates is known, 12. By drawing the bars and putting the number 12 next to the dark chocolate section only students should make the link that 12 needs to be divided by 3. Take time on the exposition and be clear. When the value of 1 part is known the number of milk, white and the total number of chocolates can be found using multiplication.

Example:



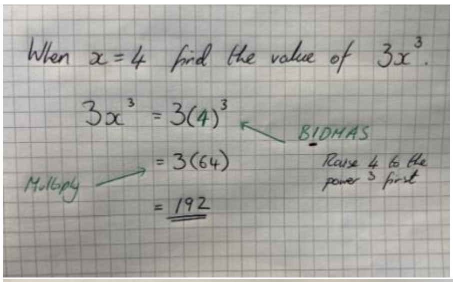
Possible Misconceptions: Students find three-part ratios difficult. Using a ratio to find one quantity when the other is known often results in students 'sharing' the known amount.

Substitution

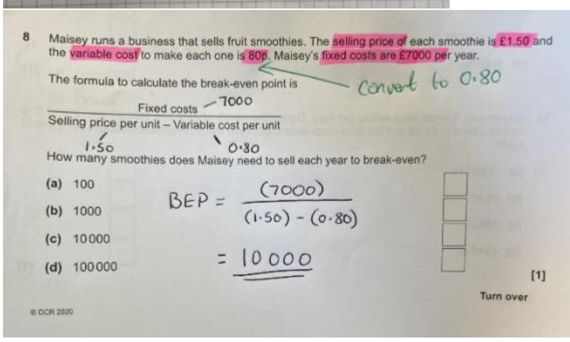
Method: Substitution is the name given to the process of replacing an algebraic letter (variable) for its value. Consider the expression 8z + 4. The answer can take on a range of values depending on what value z **actually is**. When teaching substitution, it is important at the teacher emphasizes the rules of BIDMAS are important. When substituting a value into an equation or formula it is good practice to put the numerical value in brackets, this reinforces the concept that a brackets and their coefficients are multiplied by one another as well as preventing BIDMAS errors when using a calculator*.

*If a calculator is being used the equation should be entered into the calculator as seen on the paper e.g. using the fraction button for input a fraction.

Example:



In this example without putting the value of x into brackets students may have tried to raise 34 to the power of 3.


8 Maisey runs a business that sells fruit smoothies. The selling price of each smoothie is £1.50 and the variable cost to make each one is 80p. Maisey's fixed costs are £7000 per year.

The formula to calculate the break-even point is

$$\frac{\text{Fixed costs}}{\text{Selling price per unit} - \text{Variable cost per unit}}$$

How many smoothies does Maisey need to sell each year to break-even?

(a) 100

(b) 1000

(c) 10 000

(d) 100 000

[1]

Turn over

© OCR 2020

Use of corporate language: Reinforcement of the word variable is important, so students understand the values of letters change.

Possible Misconceptions: Refrain from using a=1, b=2 etc. Especially when introducing substitution, Students tend to assign the position in the alphabet as the given value for a variable. Students may also forget the law of BIDMAS apply to algebraic expression to.

Expanding Single Brackets

Method: The method for expanding brackets should be grid method.

Example:

Answer = 3x -15

Use of corporate language: Where they are expanding and simplifying a single bracket, students must understand the meaning of the word coefficient (the term on the outside of the bracket).

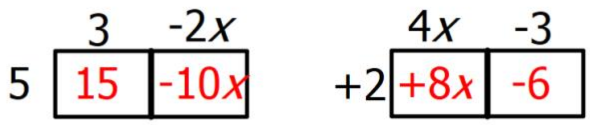
Possible Misconceptions: In an example such as 3(4y - 2) students must recognise that the second term in the brackets in "-2" and not multiply 3 by 2 an simply subtract after, this will cause misconceptions when students come to expand multiple single brackets and double brackets.

Expanding Multiple Single Brackets

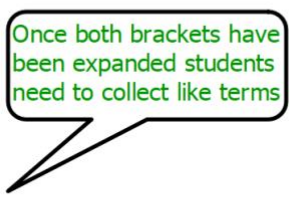
Method: The method for expanding brackets should be grid method, even for single brackets.

Example:

$$5(3 - 2x) + 2(4x - 3) =$$



$15 - 10x + 8x - 6 = 9 - 2x$



Use of corporate language: Where they are expanding and simplifying a single bracket, students must understand the meaning of the word coefficient and its meaning (the term on the outside of the bracket).

Possible Misconceptions: Where they are expanding and simplifying more than one single bracket 3(4y - 2) -3(8 - 9y) they must all be taught to see the coefficient of the second bracket as -3. Not to see it as 3 and then subtract the whole expression from the first as this leads to more errors.

Expanding Double and Triple Brackets

Method: The method for expanding brackets should be grid method, when expanding triple brackets this should be done in two stages when 2 of the brackets are expanding before being put back into a grid and multiplied by the final bracket. Students should be exposed to questions where the leading coefficient is >1 and where negative values are present.

Example:

Example

Expand and simplify $(x+4)(x+1)(x-2)$

Start by expanding one pair of brackets.

So $(x+4)(x+1)(x-2) = (x+4)(x^2-x-2)$

Then expand these two brackets

$\times$	x	$+1$
x	x^2	$+x$
-2	$-2x$	-2

$= x^2 - 2x + x - 2$

$= x^2 - x - 2$

$\times$	x^2	$-x$	-2
x	x^3	$-x^2$	$-2x$
$+4$	$+4x^2$	$-4x$	-8

$= x^3 + 4x^2 - x^2 - 4x - 2x - 8$

$= x^3 + 3x^2 - 6x - 8$

Use of corporate language: Students must understand the meaning of the word coefficient and its meaning, they must also be able to identify the leading coefficient.

Possible Misconceptions: Some pupils may think that $(x + a)^2 \equiv x^2 + a^2$ instead of (x+a)(a+x), a common misconception is for students to forget to multiply the two numerical terms (often seen in box 4), they will add or subtract the terms instead.

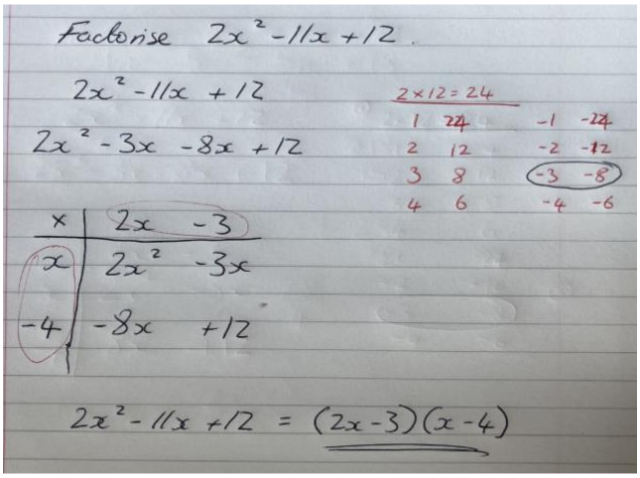
Factorising Quadratic Expressions (Double Brackets)

Method: The grid method should be used for factorising double brackets with a leading coefficient.

- Multiply the coefficient of x^2 by the constant.
- List the pairs of factors that multiply to give the result.
- Identify the pair of factors that add to the coefficient of x
- Rewrite the equation or expression replacing the single x term with the 2 terms that add to give it.
- Place the 4 terms in a grid
- Find the HCF of 2 of the terms (horizonal or vertical)
- Find the remining factors
- Place the results into double brackets.

Students should be exposed to questions where the leading coefficient is >1 and where negative values are present.

Example:



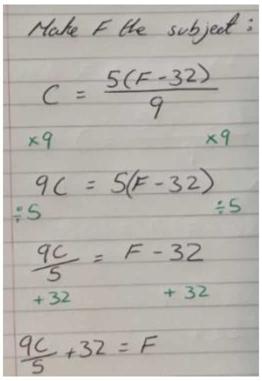
Use of corporate language: Ensure students understand the meaning of the words term, coefficient, expression, factor and factorisation.

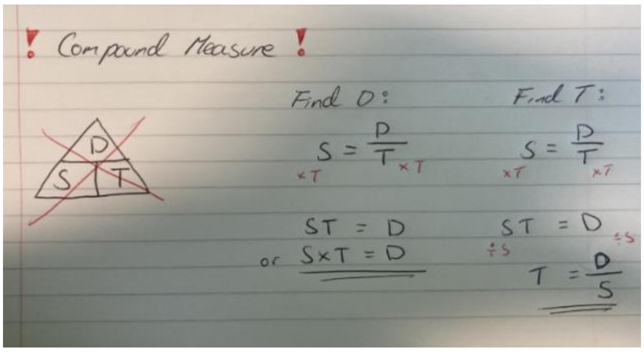
Possible Misconceptions: Students will find a factor of an expression however it may not be the HCF, students may also have issues when dividing and multiplying by negative numbers.

Rearranging Formula

Method: When changing the subject of a formula the principle of balancing (doing the same to both sides) must be used rather than a 'change side, change sign' approach. Teacher must teach students to rearrange formula rather than rely on formula triangles for compound measures or SOHCAHTOA in trigonometry. Students need to understand the various ways of writing equivalent fractional coefficients in algebra e.g. 3y/2 is the same as 3/2 multiplied by y.

Example:





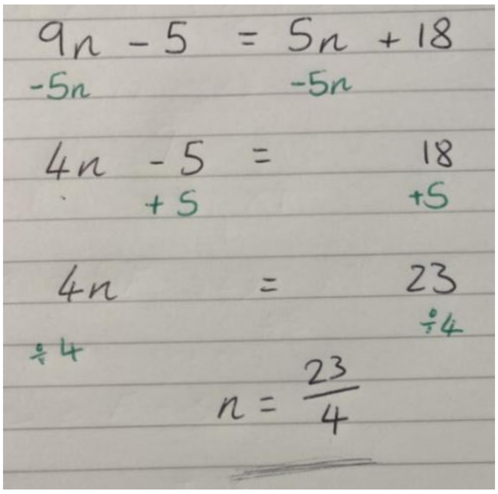
Use of corporate language: Teachers should be consistent with the use of the word balancing. Variable should be used to describe letters of unknown value.

Possible Misconceptions: Some pupils may misapply the order of operation when rearranging the equation. Students trying to apply a formula triangle learnt in a different subject area are prone to mixing them up, especially where two variables have the same notation i.e. S=D/T D=M/V.

Solving Equations

Method: When solving equations, the principle of balancing (doing the same to both sides) must be used rather than a 'change side, change sign' approach. Students must be exposed to equations that have a mix on integer and non-integer as well as positive and negative solutions. Non-integers solutions should be left as fractions. Students must understand the role of the equals sign.

Example:



Use of corporate language: Teachers should be consistent with the use of the word balancing and inverse.

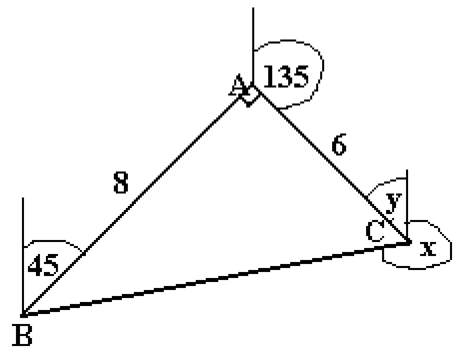
Possible Misconceptions: Some pupils may misapply the order of operation when rearranging the equation, for example when solving $2x^2=72$ they may try to square root both sides first before dividing by 2.

Bearings

Method: Bearings are always given as three figures, e.g. 025°, measured from North in a clockwise direction. Emphasise the importance of the word 'FROM'. Measure the bearing of X from Y, the letter following 'FROM' indicates where a north line should be drawn. When calculating back bearings this will always be 180 degrees difference from the outward bearing. A reference to corresponding angles when modelling should be made here.

Example: An airplane takes off from Heathrow airport, as shown in the diagram below.

The angle between the north line and the flight path of the airplane is 30∘. So we can say that the airplane is flying on a bearing of 030∘ from Heathrow airport.



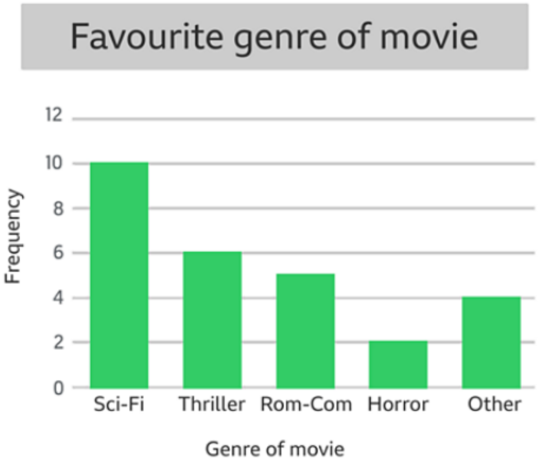
Using these types of examples in the exposition with allow for links to be make to co-interior angles and corresponding angles. To calculate the value of y students should be able to recognise the north lines are parallel and there for 135 and y are co-interior. 108-135 = 45∘

Use of corporate language:
Emphasise the importance of the word 'FROM'. Measure the bearing of X from Y

Possible Misconceptions: Students may write a 2-figure bearing and miss off the prefix of a 0. When students are asked to find the bearing ***of A from B,*** they may find the bearing ***of B from A.*** Some pupils will not appreciate the difference between angles and bearings. Some pupils will see the edge of the protractor as the destination point rather than indicating the direction of the bearing. When calculating back bearings some pupils will believe it is 180 degrees subtract the outward bearing.

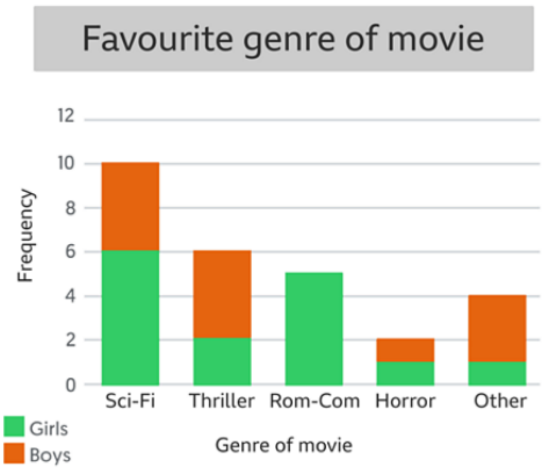
Bar Charts

Method: Students must know how to interpret and draw bar charts, composite bar charts and dual bar charts. Any drawing should be done with a pencil and ruler where appropriate. Students should be reminded that there are spaces between bars on a bar chart and that these spaces must be even- refer to the fact that a bar chart without spaces is a histogram. All bar charts should be labelled, and teachers should refer to the number of pieces of data as frequency.



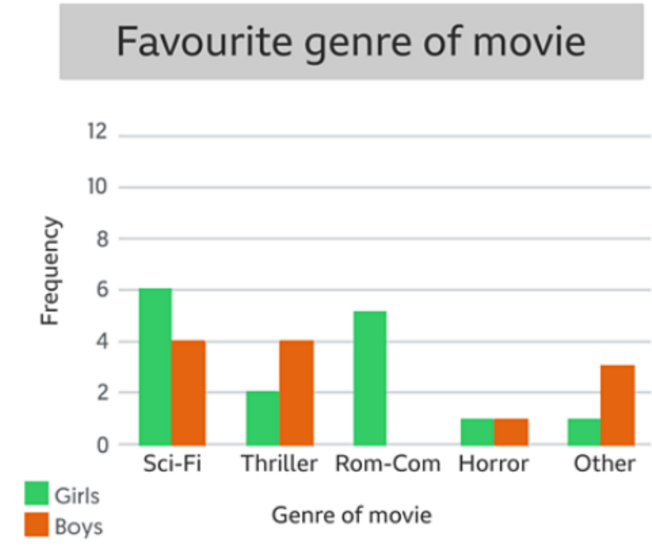
Bar charts or bar graphs are used to visually display **discrete data**. A bar chart makes it easy for the reader to pick out information about the data, such as which category has the highest **frequency**, and the **differences** between the frequencies of the categories.

Frequency = the number of times a data value occurs.



A **composite bar chart** is a chart that allows one to compare data. In a composite bar graph, or stacked bar graph, each bar is split vertically by sub-category. A key is needed to clearly show which part of the bar refers to which subcategory. A key that shows which bar represents which data is necessary to be able to read the bar chart properly, this should be emphasized to students. Also take time to show students how to read the scales carefully then interpreting a composite bar chart.

Multiple or dual bar graphs - another way of displaying the subcategories within a bar is to use a multiple bar chart, or a dual bar chart, where sub-category bars are horizontally stacked side by side. Again, a key is needed to clearly show which part of the bar refers to which subcategory.
In this example, a multiple or dual bar graph allows you to compare the gender difference within each movie category. You can clearly see that an equal proportion of boys and girls prefer horror.



It is important to note that the bars should always be of **equal width**, and there must also be an **equally spaced gap** in between each category.

As always, use a ruler and pencil when drawing bar charts and check the scale on the axes carefully.

Use of corporate language: Teachers should always refer to frequency and emphasise the importance of labelling.

Possible Misconceptions: Some pupils may not leave gaps between the bars of a bar chart. It is common for students to misread scales.

Scatter Graphs

Method: Scatter diagrams show the relationship between two variables. By looking at the diagram you can see whether there is a link between variables. If there is a link it is called correlation. Students need to be able to plot points on a scatter graph, describe a relationship between two variables and use the graph to estimate values. Teachers also need to ensure students know what is meant by the term outlier and how to identify them.

Example: The English and Maths results of ten classmates are shown in the table below:

	Sal	Kim	Bill	Tom	Gita	Alex	Bev	Ken	Alan	Jo
Maths mark	20	71	60	52	80	32	47	90	49	80
English mark	30	80	65	50	81	38	40	87	55	70

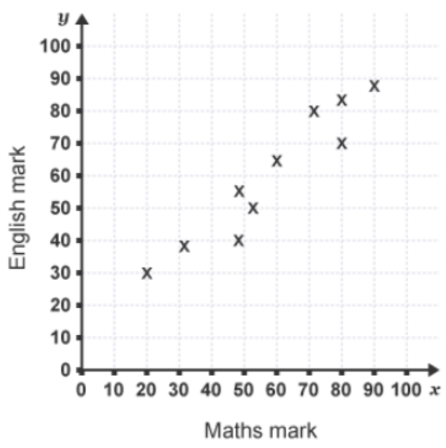


Diagram showing Maths and English marks.

To see whether there is a correlation between the Maths marks and the English marks, you can plot a scatter diagram.

The Maths mark is on the horizontal scale and the corresponding English mark on the vertical scale.

Bill's Maths mark was 60 and his English mark was 65, so his results are represented by the purple point at coordinates (60, 65).

The scatter graphs show positive correlation, the higher the maths mark the higher the English mark tends to be.

Use of corporate language: Emphasise the point that correlation describes the relationship between 2 variables. They should be confident using the word correlation in an explanation.

Possible Misconceptions: When a question does not explicitly ask a student to draw a line of best fit some students try to estimate using just the graph, this can cause them to get the answer wrong. Insist they draw a line of best fit.

Dispersion Graphs

Method: A dispersion graph shows the range of a set of data and illustrates whether data groups or is dispersed. It is a useful way of comparing sets of data. Values are plotted on the vertical axis.

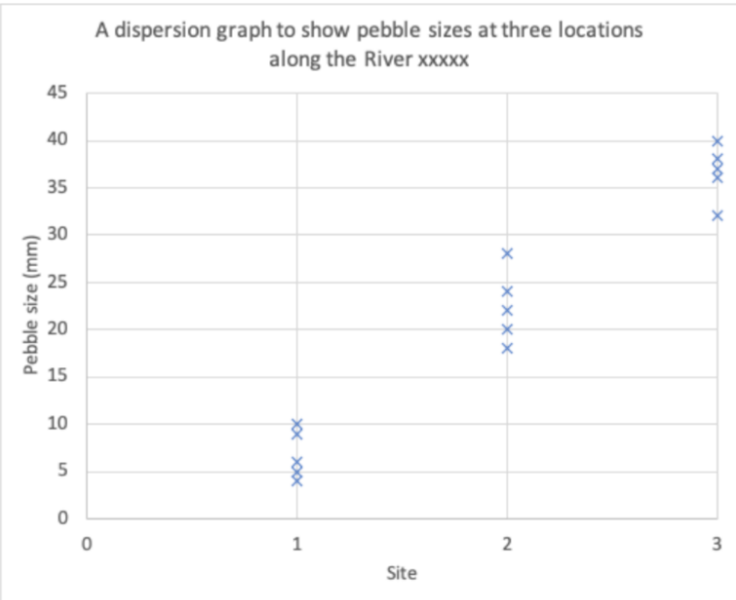
WHEN IS USING A DISPERSION GRAPH APPROPRIATE?

Dispersion graphs are ideal when you want to compare sets of data and can be used to present where the UQ and LQ are, as well as the mean, median, mode and extreme values and interquartile range.

	A	B	C	D	E	F
1		Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
2	Site 1	5	9	6	10	4
3	Site 2	18	20	24	22	28
4	Site 3	32	37	36	38	40

Dispersion graph data

Example: The dispersion graph below shows the size of pebbles at three sites along a river. The data below will be plotted on a dispersion graph. The pebbles have been measured in mm.



Dispersion graph for pebbles sampled at three sites

Use of corporate language: Read the title to see what the graph is showing. Ensure you understand what each axis represents. Identify outliers (anomalies in the data). Investigate the patterns show on the graph. Complete statistical analysis e.g. what is the mean (you could then plot this in a different colour), what is the range?, What is the media? What is the interquartile range? etc.

Range and Spread

Range

Method: It's very easy to calculate the **range** of a set of data by subtracting the smallest value from the largest.

A range is important to us because it tells us something about the **spread** of the data and when we have large group of numbers to handle this can be a quick way of saying something about the data set as a whole.

Note Outliers: One limitation of the range is that it is affected by outliers. Consider the data below:

1, 1, 3, 12, 2, 4, 5, 2, 1, 1, 6, 3, 4

Currently the range is 11 (12-1). However, this does not tell us anything useful about the data. The reason for this is that the one large value (12) is distorting the range. Without this one value the range would be 5 (6-1). The range has been more than halved. There are **other measures of spread** that are not so affected by outliers.

Example: Calculate the **range** of a set of data by subtracting the smallest value from the largest.

3, 5, 7, 8, 9, 13, 15

The range is 15 - 3 = 12.

Median

Method: The median of a set of data is the **middle value** when the data is arranged in size order. When two middle values are present the median is the mean of the two.

From the above data used in calculating range:

3, 5, 7, 8, 9, 13, 15

There are seven values, so the middle value will be the fourth. The median of the above data is therefore eight.

Sometimes it is said that if we have n pieces of data, the median is the (n+1)2 value. Using the above example again we see that we have seven values (n=7) so the (n+1)2 value would be (7+1)2 = the 4th value, which as mentioned previously is eight.

Example: Let's look at another example, what is the median of the following data?

12, 14, 13, 20, 17, 15

First we must arrange this into size order:

12, 13, 14, 15, 17, 20

Now we notice that there are two **middle numbers** (14 and 15) so we must find the mean of these. The mean of two numbers is simply the number halfway between them [(14+15)2].

So the median is 14.5.

Use of corporate language: The median is a measure of **central tendency**. It tells us something valuable about the data – roughly what values we can expect in the middle. The mean is another measure of central tendency, but like the range it can be affected by outliers or extreme values.

Quartiles and Interquartile Range

Method: The interquartile range is another measure of spread, except that it has the added advantage of not being affected by large outlying values.

In order to calculate this value we must first understand what the lower quartile, median and upper quartile are:

- the lower quartile is the median of the lower half of the data. The (n+1)4 value
- the upper quartile is the median of the upper half of the data. The 3(n+1)4 value

Histograms

Method: Histograms are a way of representing data they are 'diagrams made of rectangles whose areas are proportional to the frequency of the group'. They are like bar charts, but show the frequency density instead of the frequency, it is the **area of the bar**, and not the height, shows the frequency of the data. The y-axis shows the frequency density of a class width.

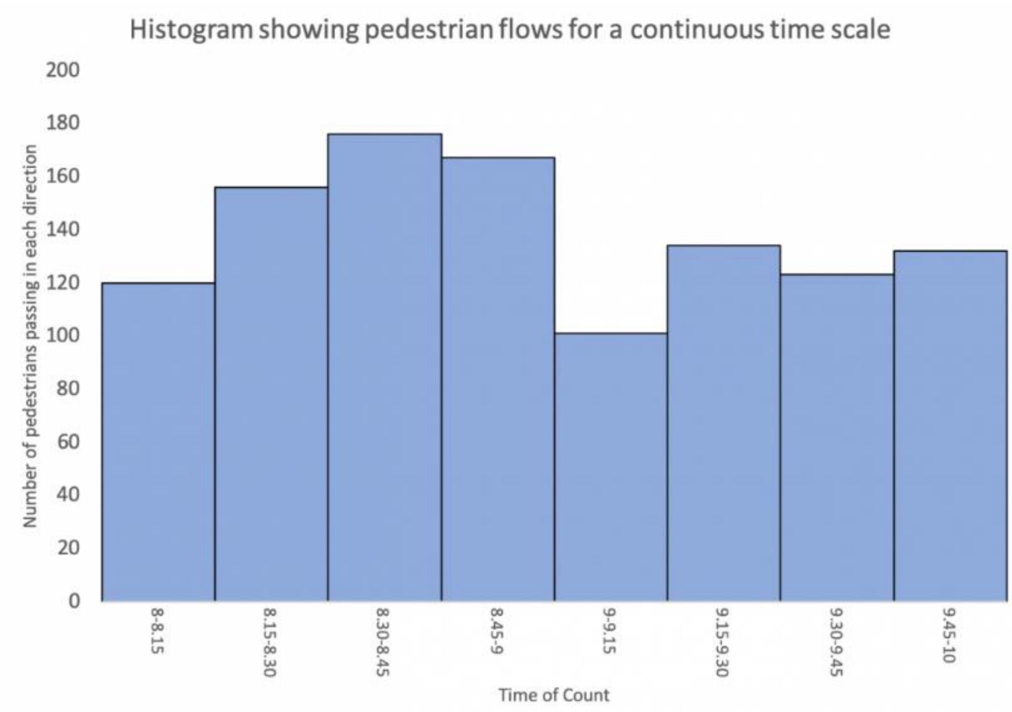
They can be used to determine information about the **distribution of data** as Histograms are typically used when the data is in groups of unequal width.

Example: Equal Class Width

Histograms are ideal for presenting continuous data. Continuous data is data that falls in a continuous sequence e.g. time, distance, and temperature. An example of this would be after counting pedestrians at 15-minute intervals over 2 hours, a histogram could be used to present the results.

Creating a histogram: In this example, we are going to produce a histogram to show the results of a pedestrian count completed at 15-minute intervals over a continuous period of time. Students have collected raw data that shows the number of pedestrians that passed them during 15-minute intervals over two hours.

- 8-8.15 am – 120
- 8.15-8.30 am – 156
- 8.30-8.45 am – 176
- 8.45-9 am – 167
- 9-9.15 am – 101
- 9.15-9.30 am – 134
- 9.30-9.45 am – 123
- 9.45-10 am – 132



Step 1 – Decide on the scale of the X-axis
Decide on an appropriate scale on the X-axis for the bars. The bars should be the same width and there should be no gaps between the bars.

Step 2 – Decide on the scale of the Y-axis
Decide on a suitable scale for the Y-axis for the number of pedestrians. The scale should be spaced evenly and allow for the highest number in the data set to be included.

Step 3 – Create the histogram
Accurately draw the bars for each piece of data. As the data is continuous, each bar should be shaded in the same colour

Step 4 – Finish your graph
Include a title and label each axis.

Reading a Histogram: To read a bar chart, read along the x-axis (bottom) to find the bar you want to read. Go to the top of the bar and read across to the scale on the y-axis to work out the value. Using a ruler can help with this.